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TRANSLATION OF BIPOLAR VALUED FUZZY SUBHEMIRING OF A HEMIRING

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ABSTRACT

In this paper, some definitions and new Theorems of a bipolar valued fuzzy subhemiring of a hemiring are presented. Using the definition of translation of bipolar valued fuzzy subhemiring of a hemiring, union, intersection and translation Theorems are introduced.

KEYWORDS: *Bipolar Valued Fuzzy Subset, Image, Preimage, Bipolar Valued Fuzzy Subhemiring, Translation*

INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [15]. Since its inception, the theory of fuzzy sets has developed in many directions and is finding applications in a wide variety of fields. In [5] Rosenfeld used this concept to develop the theory of fuzzy groups of a group. In fact, many basic properties in group theory are found to be carried over to fuzzy groups. Lee [9] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Anitha.M.S., Muruganantha Prasad & K. Arjunan[1] defined as bipolar valued fuzzy subgroups of a group. In this paper, we introduce the concept of bipolar valued fuzzy translation of bipolar valued fuzzy subhemirings of a hemiring. Using these concepts, some results are established.

1. PRELIMINARIES

1.1. Definition

A bipolar valued fuzzy set (BVFS) of X is defined as an object of the form $A = \{ < x, A^{+}(x), A^{-}(x) >/ x\in X\}$, where $A^{+} : X\rightarrow [0, 1]$ and $A^{-} : X\rightarrow [-1, 0]$. The positive membership degree $A^{+}(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^{-}(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

1.2. Example

A = { < x, 0.8, –0.6 >, < y, 0.7, –0.5 >, < z, 0.9, –0.4 >} is a bipolar valued fuzzy subset of X = {x, y, z }.

1.3. Definition

Let S be a hemiring. A bipolar valued fuzzy subset B of S is said to be a bipolar valued fuzzy subhemiring of S (BVFSHR) if the following conditions are satisfied,

- $B^+(x+y) \geq \min\{ B^+(x), B^+(y) \}$
- $B^+(xy) \geq \min\{ B^+(x), B^+(y) \}$
- $B^-(x+y) \leq \max\{ B^-(x), B^-(y) \}$
- $B^-(xy) \leq \max\{ B^-(x), B^-(y) \}$ for all x and y in S.

1.4. Example:

Let S = Z_3 = { 0, 1, 2 } be a hemiring with respect to the ordinary addition and multiplication. Then A = { < 0, 0.8, – 0.9 >, < 1, 0.6, –0.8 >, < 2, 0.6, – 0.8 > } is a bipolar valued fuzzy subhemiring of S.

1.5. Definition

Let X and Y be any two sets. Let f : X→ Y be any function and let A be a bipolar valued fuzzy subset in X, V be a bipolar valued fuzzy subset in f(X) = Y, defined by $V^+(y) = \sup_{x\in f^{-1}(y)} A^+(x)$ and $V^-(y) = \inf_{x\in f^{-1}(y)} A^-(x)$, for all x in X and y in Y. A is called a preimage of V under f and is defined as $A^+(x) = V^+(f(x))$, $A^-(x) = V^-(f(x))$ for all x in X and is denoted by $f^{-1}(V)$.

1.6. Definition

Let A = $\langle A^+, A^- \rangle$ be a bipolar valued fuzzy subset of X and α in [0, 1– sup { $A^+(x)$}], β in [–1– inf { $A^-(x)$}, 0]. Then T = $\langle T^+, T^- \rangle$ is called a bipolar valued fuzzy translation of A if $T^+(x) = T^{+A}{}_{\alpha_i}(x) = A^+(x)+ \alpha$, $T^-(x) = T_\beta^{-A}(x) = A^-(x) + \beta$, for all x in X.

1.7 Example

Consider the set X = { 0, 1, 2, 3, 4 }. Let A = { (0, 0.5, –0.1), (1, 0.4, –0.3), (2, 0.6, –0.05), (3, 0.45, –0.2), (4, 0.2, –0.5) } be a bipolar valued fuzzy subset of X and α = 0.1, β = –0.1. Then the bipolar valued fuzzy translation of A is T = $T^A_{(0.1, -0.1)}$ = { (0, 0.6, –0.2), (1, 0.5, –0.4), (2, 0.7, –0.15), (3, 0.55, –0.3), (4, 0.3, –0.6) }.

2. PROPERTIES

2.1. Theorem

If M and N are two bipolar valued fuzzy translations of bipolar valued fuzzy subhemiring A of a hemiring R, then their intersection M∩N is also a bipolar valued fuzzy translation of A.

Proof: Let x and y belong to R. Let M = $T^{A}_{(\alpha,\beta)}$ = { ⟨ x, $A^{+}(x) + \alpha$, $A^{-}(x) + \beta$ ⟩ / x∈R } and N = $T^{A}_{(\gamma,\delta)}$ = { ⟨ x, $A^{+}(x) + \gamma$, $A^{-}(x) + \delta$ ⟩ / x∈R } be two bipolar valued fuzzy translations of bipolar valued fuzzy subhemiring A = ⟨ A^{+}, A^{-} ⟩ of R. Let C = M∩N and C = { ⟨ x, $C^{+}(x)$, $C^{-}(x)$ ⟩ / x∈R }, where $C^{+}(x)$ = min { $A^{+}(x) + \alpha$, $A^{+}(x) + \gamma$ } and $C^{-}(x)$ = max { $A^{-}(x) + \beta$, $A^{-}(x)+\delta$ }.

Case (i): $\alpha \leq \gamma$ and $\beta \leq \delta$. Now $C^{+}(x)$ = min{$M^{+}(x)$, $N^{+}(x)$ } = min{$A^{+}(x)+\alpha$, $A^{+}(x)+\gamma$}= $A^{+}(x)+\alpha$ = $M^{+}(x)$ for all x in R. And $C^{-}(x)$ = max { $M^{-}(x)$, $N^{-}(x)$ } = max { $A^{-}(x) + \beta$, $A^{-}(x) + \delta$ }= $A^{-}(x) + \delta$ = $N^{-}(x)$ for all x in R. Therefore C = $T^{A}_{(\alpha,\delta)}$ = { ⟨ x, $A^{+}(x) + \alpha$, $A^{-}(x)+ \delta$ ⟩ / x∈R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring A of R.

Case (ii): $\alpha \geq \gamma$ and $\beta \geq \delta$. Now $C^{+}(x)$ = min { $M^{+}(x)$, $N^{+}(x)$ } = min { $A^{+}(x) + \alpha$, $A^{+}(x)+ \gamma$ } = $A^{+}(x) + \gamma$ = $N^{+}(x)$ for all x in R. And $C^{-}(x)$ = max { $M^{-}(x)$, $N^{-}(x)$ } = max { $A^{-}(x) + \beta$, $A^{-}(x) + \delta$ } = $A^{-}(x) + \beta$ = $M^{-}(x)$ for all x in R. Therefore C = $T^{A}_{(\gamma,\beta)}$ = { ⟨ x, $A^{+}(x)+ \gamma$, $A^{-}(x)+ \beta$ ⟩ / x∈R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring A of R.

Case (iii): $\alpha \leq \gamma$ and $\beta \geq \delta$. Clearly C = $T^{A}_{(\alpha,\beta)}$ = { ⟨ x, $A^{+}(x) + \alpha$, $A^{-}(x)+\beta$ ⟩ / x∈R} is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring A of R.

Case (iv): $\alpha \geq \gamma$ and $\beta \leq \delta$. Clearly C = $T^{A}_{(\gamma,\delta)}$ = { ⟨ x, $A^{+}(x)+ \gamma$, $A^{-}(x)+ \delta$ ⟩ / x∈R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring A of R. In other cases are true, so in all the cases, the intersection of any two bipolar valued fuzzy translations of bipolar valued fuzzy subhemiring A of R is a bipolar valued fuzzy translation of A.

2.2. Theorem

The intersection of a family of bipolar valued fuzzy translations of bipolar valued fuzzy subhemiring A of a hemiring R is a bipolar valued fuzzy translation of A.

Proof: Using the Theorem 2.1, we can prove easily.

2.3. Theorem

Union of any two bipolar valued fuzzy translations of bipolar valued fuzzy subhemiring A of a hemiring R is a bipolar valued fuzzy translation of A.

Proof: Let x and y belong to R. Let M = $T^{A}_{(\alpha,\beta)}$ = { ⟨ x, $A^{+}(x) + \alpha$, $A^{-}(x) + \beta$ ⟩ / x∈R } and N = $T^{A}_{(\gamma,\delta)}$ = { ⟨ x, $A^{+}(x) + \gamma$, $A^{-}(x) + \delta$ ⟩ / x∈R } be two bipolar valued fuzzy translations of bipolar valued fuzzy subhemiring A = ⟨ A^{+}, A^{-} ⟩ of R. Let C = M∪N and C = { ⟨ x, $C^{+}(x)$, $C^{-}(x)$ ⟩ / x∈R }, where $C^{+}(x)$ = max { $A^{+}(x) + \alpha$, $A^{+}(x) + \gamma$ } and $C^{-}(x)$ = Min { $A^{-}(x)+ \beta$, $A^{-}(x)+ \delta$ }.

Case (i): $\alpha \leq \gamma$ and $\beta \leq \delta$. Now $C^{+}(x)$ = max { $M^{+}(x)$, $N^{+}(x)$ } = max{$A^{+}(x)+\alpha$, $A^{+}(x)+ \gamma$}= $A^{+}(x)+ \gamma$ = $N^{+}(x)$ for all x and y in R. And $C^{-}(x)$ = Min { $M^{-}(x)$, $N^{-}(x)$ } = Min { $A^{-}(x)+ \beta$, $A^{-}(x)+ \delta$} = $A^{-}(x)+ \beta$ = $M^{-}(x)$ for all x in R.

Therefore C = $T^{A}_{(\gamma,\beta)}$ = { ⟨ x, $A^+(x) + \gamma$, $A^-(x) + \beta$ ⟩ / x∈R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring A of R.

Case (ii): $\alpha \geq \gamma$ and $\beta \geq \delta$. Now $C^+(x)$ = max { $M^+(x)$, $N^+(x)$ } = max { $A^+(x) + \alpha$, $A^+(x) + \gamma$}= $A^+(x) + \alpha = M^+(x)$ for all x in R. And $C^-(x)$ = min{ $M^-(x)$, $N^-(x)$ } = min {$A^-(x)+\beta$, $A^-(x)+ \delta$} = $A^-(x)+ \delta = N^-(x)$ for all x in R. Therefore C = $T^{A}_{(\alpha,\delta)}$ = { ⟨ x, $A^+(x)+ \alpha$, $A^-(x)+ \delta$ ⟩ / x∈R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring A of R.

Case (iii): $\alpha \leq \gamma$ and $\beta \geq \delta$. Clearly C = $T^{A}_{(\gamma,\delta)}$ = { ⟨ x, $A^+(x) + \gamma$, $A^-(x)+\delta$ ⟩ / x∈R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring A of R.

Case (iv): $\alpha \geq \gamma$ and $\beta \leq \delta$. Clearly C = $T^{A}_{(\alpha,\beta)}$ = { ⟨ x, $A^+(x) + \alpha$, $A^-(x)+\beta$⟩ / x∈R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring A of R. In other cases are true, so in all the cases, union of any two bipolar valued fuzzy translations of bipolar valued fuzzy subhemiring A of R is a bipolar valued fuzzy translation of A.

2.4. Theorem

The union of a family of bipolar valued fuzzy translations of bipolar valued fuzzy subhemiring A of a hemiring R is a bipolar valued fuzzy translation of A.

Proof: Using the Theorem 2.1, we can prove easily.

2.5. Theorem

A bipolar valued fuzzy translation of a bipolar valued fuzzy subhemiring A of a hemiring R is a bipolar valued fuzzy subhemiring of R.

Proof: Assume that T = ⟨ T^+, T^- ⟩ is a bipolar valued fuzzy translation of a bipolar valued fuzzy subhemiring A = ⟨ A^+, A^- ⟩ of a hemiring R. Let x and y in R. We have $T^+(x+y) = A^+(x+y) + \alpha \geq$ min{ $A^+(x)$, $A^+(y)$ } + α = min { $A^+(x) + \alpha$, $A^+(y) + \alpha$ } = min { $T^+(x)$, $T^+(y)$ }. Therefore $T^+(x+y) \geq$ min { $T^+(x)$, $T^+(y)$ } for all x and y in R. And $T^+(xy) = A^+(xy) + \alpha$ $\geq$ min{ $A^+(x)$, $A^+(y)$ } + α = min { $A^+(x) + \alpha$, $A^+(y) + \alpha$ }= min { $T^+(x)$, $T^+(y)$ }. Therefore $T^+(xy) \geq$ min { $T^+(x)$, $T^+(y)$ } for all x and y in R. Also $T^-(x+y) = A^-(x+y) +\beta \leq$ max { $A^-(x)$, $A^-(y)$ }+ β = max { $A^-(x) + \beta$, $A^-(y) + \beta$ } = max { $T^-(x)$, $T^-(y)$ }. Therefore $T^-(x+y) \leq$ max{ $T^-(x)$, $T^-(y)$ } for all x and y in R. And $T^-(xy) = A^-(xy) +\beta \leq$ max{ $A^-(x)$, $A^-(y)$ }+ β = max { $A^-(x) + \beta$, $A^-(y) + \beta$ }= max { $T^-(x)$, $T^-(y)$ }. Therefore $T^-(xy) \leq$ max { $T^-(x)$, $T^-(y)$ } for all x and y in R. Hence T is a bipolar valued fuzzy subhemiring of R.

2.6. Theorem

Let (R, +, .) and (R^I, +, .) be any two hemirings and f be a homomorphism. Then the homomorphic image of a bipolar valued fuzzy translation of a bipolar valued fuzzy subhemiring A of R is also a bipolar valued fuzzy subhemiring of R^I.

Proof: Let V = (V^+, V^-) = f($T^A_{(\alpha,\beta)}$), where $T^A_{(\alpha,\beta)}$ is a bipolar valued fuzzy translation of a bipolar valued fuzzy subhemiring A = (A^+, A^-) of R. We have to prove that V is a bipolar valued fuzzy subhemiring of R^I. For all f(x) and f(y) in R^I, we have V^+[f(x)+f(y)] = V^+ [f(x+y)] ≥ $T^{+A}{}_\alpha$(x+y) = A^+(x+y)+ α ≥ min{ A^+(x), A^+(y)}+ α = min{A^+(x)+α, A^+(y)+α} = min{ $T^{+A}{}_\alpha$(x), $T^{+A}{}_\alpha$(y) }

which implies that V^+[f(x)+f(y)] ≥ min { V^+(f(x)), V^+(f(y)) } for all f(x) and f(y) in R^I. And V^+[f(x)f(y)] = V^+ [f(xy)] ≥ $T^{+A}{}_\alpha$(xy) = A^+(xy)+ α ≥ min{ A^+(x), A^+(y) }+α = min{A^+(x)+α, A^+(y)+α }= min{ $T^{+A}{}_\alpha$(x), $T^{+A}{}_\alpha$(y) }

which implies that V^+[f(x)f(y)] ≥ min { V^+(f(x)), V^+(f(y)) } for all f(x) and f(y) in R^I. Also V^-[f(x)+f(y)] = V^-[f(x+y)] ≤ T^{-A}_β(x+y) = A^-(x+y)+ β ≤ max{ A^-(x), A^-(y) }+β = max { A^-(x)+β, A^-(y)+β} = max { T^{-A}_β(x), T^{-A}_β(y) }

which implies that V^-[f(x)+f(y)] ≤ max { V^-(f(x)), V^-(f(y)) } for all f(x) and f(y) in R^I. And V^-[f(x)f(y)] = V^-[f(xy)] ≤ T^{-A}_β(xy) = A^-(xy)+ β ≤ max {A^-(x), A^-(y) }+ β = max { A^-(x)+β, A^-(y)+β} = max { T^{-A}_β(x), T^{-A}_β(y) } which implies that V^-[f(x)f(y)] ≤ max { V^-(f(x)), V^-(f(y)) } for all f(x) and f(y) in R^I. Therefore V is a bipolar valued fuzzy subhemiring of R^I.

2.7. Theorem

Let (R, +, .) and (R^I, +, .) be any two hemirings and f be a homomorphism. Then the homomorphic pre-image of bipolar valued fuzzy translation of a bipolar valued fuzzy subhemiring V of R^I is a bipolar valued fuzzy subhemiring of R.

Proof: Let T = $T^V_{(\alpha,\beta)}$ = f(A), where $T^V_{(\alpha,\beta)}$ is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring V = (V^+, V^-) of R^I. We have to prove that A = (A^+, A^-) is a bipolar valued fuzzy subhemiring of R. Let x and y in R. Then A^+(x+y) = T^{+V}_α (f(x+y)) = T^{+V}_α (f(x)+f(y)) = V^+[f(x)+f(y)]+ α ≥ min { V^+(f(x)), V^+(f(y)) }+ α = min {V^+(f(x)) + α, V^+(f(y)) + α } = min { T^{+V}_α (f(x)), T^{+V}_α (f(y))} = min { A^+(x), A^+(y) } which implies that A^+(x+y) ≥ min{A^+(x), A^+(y)} for all x, y in R. And A^+(xy) = T^{+V}_α (f(xy)) = T^{+V}_α (f(x)f(y)) = V^+[f(x)f(y)] + α ≥ min { V^+(f(x)), V^+(f(y)) }+ α = min { V^+(f(x)) + α, V^+(f(y)) + α } = min { T^{+V}_α (f(x)), T^{+V}_α (f(y)) } = min { A^+(x), A^+(y) } which implies that A^+(xy) ≥ Min { A^+(x), A^+(y) } for all x and y in R. Also A^-(x+y) = T^{-V}_β (f(x+y)) = T^{-V}_β (f(x)+f(y)) = V^-[f(x)+f(y)] +β ≤ max {V^-(f(x)), V^-(f(y)) }+β = max{V^-(f(x))+β, V^-(f(y)) + β } = max{ T^{-V}_β(f(x)), T^{-V}_β (f(y)) } = max { A^-(x), A^-(y) } which implies A^-(x+y) ≤ max{ A^-(x), A^-(y) } for all x and y in R. And A^-(xy) = T^{-V}_β (f(xy)) = T^{-V}_β(f(x)f(y)) = V^-[f(x)f(y)] +β ≤ max { V^-(f(x)), V^-(f(y)) }+ β = max { V^-(f(x)) + β, V^-(f(y)) + β } = max { T^{-V}_β (f(x)), T^{-V}_β(f(y)) } = max { A^-(x), A^-(y) } which implies A^-(xy) ≤ max{ A^-(x), A^-(y) } for all x and y in R. Therefore A is a bipolar valued fuzzy subhemiring of R.

2.8. Theorem

Let (R, +, .) and (R^{l}, +, .) be any two hemirings and f be a anti-homomorphism. Then the anti-homomorphic image of a bipolar valued fuzzy translation of a bipolar valued fuzzy subhemiring A of R is also a bipolar valued fuzzy subhemiring of R^{l}.

Proof: Let V = (V^{+}, V^{-}) = f($T^{A}_{(\alpha,\beta)}$), where $T^{A}_{(\alpha,\beta)}$ is a bipolar valued fuzzy translation of a bipolar valued fuzzy subhemiring A = (A^{+}, A^{-}) of R. We have to prove that V is a bipolar valued fuzzy subhemiring of R^{l}. For all f(x) and f(y) in R^{l}, we have V^{+}[f(x)+f(y)] = V^{+} [f(y+x)] ≥ $T^{+A}{}_{\alpha}$(y+x) = A^{+}(y+x) + α ≥ min{ A^{+}(y), A^{+}(x) }+ α = min {A^{+}(x)+α, A^{+}(y)+α } = min { $T^{+A}{}_{\alpha}$(x), $T^{+A}{}_{\alpha}$(y) } which implies that V^{+}[f(x)+f(y)] ≥ min { V^{+}(f(x)), V^{+}(f(y)) } for all f(x) and f(y) in R^{l}. And V^{+}[f(x)f(y)] = V^{+} [f(yx)] ≥ $T^{+A}{}_{\alpha}$(yx) = A^{+}(yx) + α ≥ min{ A^{+}(y), A^{+}(x) }+α = min {A^{+}(x)+α, A^{+}(y)+α } = min { $T^{+A}{}_{\alpha}$(x), $T^{+A}{}_{\alpha}$(y) } which implies that V^{+}[f(x)f(y)] ≥ min { V^{+}(f(x)), V^{+}(f(y)) } for all f(x) and f(y) in R^{l}. Also V^{-}[f(x)+f(y)] = V^{-}[f(y+x)] ≤ T_{β}^{-A}(y+x) = A^{-}(y+x) + β ≤ max { A^{-}(y), A^{-}(x) }+ β = max { A^{-}(x)+ β, A^{-}(y) + β }= max { T_{β}^{-A}(x), T_{β}^{-A}(y) } which implies that V^{-}[f(x) + f(y)] ≤ max { V^{-}(f(x)), V^{-}(f(y)) } for all f(x) and f(y) in R^{l}. And V^{-}[f(x)f(y)] = V^{-}[f(yx)] ≤ T_{β}^{-A}(yx) = A^{-}(yx) + β ≤ max { A^{-}(x), A^{-}(y) } + β = max { A^{-}(x) + β, A^{-}(y) + β}= max { T_{β}^{-A}(x), T_{β}^{-A}(y) } which implies that V^{-}[f(x)f(y)] ≤ max { V^{-}(f(x)), V^{-}(f(y)) } for all f(x) and f(y) in R^{l}. Therefore V is a bipolar valued fuzzy subhemiring of R^{l}.

2.9. Theorem

Let (R, +, .) and (R^{l}, +, .) be any two hemirings and f be an anti-homomorphism. Then the anti-homomorphic pre-image of bipolar valued fuzzy translation of a bipolar valued fuzzy subhemiring V of R^{l} is a bipolar valued fuzzy subhemiring of R.

Proof: Let T = $T^{V}_{(\alpha,\beta)}$ = f(A), where $T^{V}_{(\alpha,\beta)}$ is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring V = (V^{+}, V^{-}) of R^{l}. We have to prove that A = (A^{+}, A^{-}) is a bipolar valued fuzzy subhemiring of R. Let x and y in R. Then A^{+}(x+y) = T_{α}^{+V} (f(x+y)) = T_{α}^{+V} (f(y)+f(x)) = V^{+}[f(y)+f(x)] + α ≥ min { V^{+}(f(y)), V^{+}(f(x)) }+ α = min { V^{+}(f(x)) + α, V^{+}(f(y)) + α } = min{ T_{α}^{+V} (f(x)), T_{α}^{+V} (f(y))} = min{A^{+}(x), A^{+}(y) } which implies that A^{+}(x+y) ≥ min { A^{+}(x), A^{+}(y) } for all x, y in R. And A^{+}(xy) = T_{α}^{+V} (f(xy)) = T_{α}^{+V} (f(y)f(x)) = V^{+}[f(y)f(x)] + α ≥ min { V^{+}(f(y)), V^{+}(f(x)) }+ α = min { V^{+}(f(x)) + α, V^{+}(f(y))+ α } = min { T_{α}^{+V} (f(x)), T_{α}^{+V} (f(y)) } = min { A^{+}(x), A^{+}(y) } which implies that A^{+}(xy) ≥ min { A^{+}(x), A^{+}(y) } for all x and y in R. Also A^{-}(x+y) = T_{β}^{-V} (f(x+y)) = T_{β}^{-V} (f(y)+f(x)) = V^{-}[f(y)+f(x)] +β ≤ max { V^{-}(f(y)), V^{-}(f(x)) } + β = max { V^{-}(f(x)) + β, V^{-}(f(y)) + β } = max { T_{β}^{-V} (f(x)), T_{β}^{-V} (f(y)) } = max { A^{-}(x), A^{-}(y) } which implies A^{-}(x+y) ≤ max{ A^{-}(x), A^{-}(y) } for all x and y in R. And A^{-}(xy) = T_{β}^{-V} (f(xy)) = T_{β}^{-V} (f(y)f(x)) = V^{-}[f(y)f(x)] +β ≤ max { V^{-}(f(y)), V^{-}(f(x)) } + β = max { V^{-}(f(x)) + β, V^{-}(f(y)) + β } = max { T_{β}^{-V} (

f(x)), T_{β}^{-V} (f(y)) } = max { A^{-}(x), A^{-}(y) } which implies A^{-}(xy) ≤ max{ A^{-}(x), A^{-}(y) } for all x and y in R. Therefore A is a bipolar valued fuzzy subhemiring of R.

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